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# Family Symmetries and Proton Decay\*

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The proton decay modes  $p \rightarrow K^0 e^+$  and  $p \rightarrow K^0 \mu^+$  may be visible in certain supersymmetric theories, and if seen would provide evidence for new flavor physics at extremely short distances. These decay modes can arise from the dimension five operator  $(Q_1 Q_1 Q_2 L_{1,2})$ , where  $Q_i$  and  $L_i$  are  $i^{th}$  generation quark and lepton superfields respectively. Such an operator is not generated at observable levels due to gauge or Higgs boson exchange in a minimal GUT. However in theories that explain the fermion mass hierarchy, it may be generated at the Planck scale with a strength such that the decays  $p \rightarrow K^0 \ell^+$  are both compatible with the proton lifetime and visible at Super-Kamiokande. Observable proton decay can even occur in theories without unification.

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## 1. Introduction

There is an  $SU(3)^5$  chiral flavor symmetry in the standard model in the limit that the Yukawa couplings vanish. It is possible that the Yukawa couplings are themselves the fundamental parameters that break these symmetries, but their hierarchical structure suggests that they are fossils left over from a simpler form of flavor symmetry violation at short distances. It is therefore interesting to examine flavor changing processes for effects that would not occur if the Yukawa couplings were the sole source of flavor violation. Since the Yukawa couplings of the first two families are very small, flavor changing processes involving the first two families are sensitive tests for the existence of new flavor physics at some scale  $\Lambda$  that violates the first family chiral symmetries directly. In the standard model, flavor changing operators are suppressed by either  $1/\Lambda$  in the case of neutrino masses, or  $1/\Lambda^2$  for four fermion operators leading to  $B$  violation, rare decays and flavor changing neutral currents (FCNC). In the supersymmetric standard model the power counting is different due to the existence of squarks. There are new sources of flavor symmetry violation including dimension two squark masses, and both dimension four and five  $B$  and  $L$  violating operators. The first two sources are problematic for supersymmetry and must be eliminated; the dimension four  $B$  and  $L$  violating operators may be forbidden by imposing a symmetry, such as either  $R$ -parity or a flavor symmetry, while the lack of observed FCNC suggests the existence of some kind of flavor symmetry for the squark mass matrix.

In this Letter we focus on the dimension five  $B$  and  $L$  violating operators in supersymmetric (SUSY) theories, which allow one to examine flavor physics at extremely short distances. It may not be obvious why such operators contain information about flavor. The effective  $B$ -violating operators in the original (non-supersymmetric) grand unified theories (GUTs) are four-fermi operators of quark and lepton fields induced by an exchange of GUT-heavy gauge boson (for a review, see [1]). Therefore the structure of the operators are determined solely by the gauge quantum numbers of the fields under the unified group. An observation of proton decay, such as the mode  $p \rightarrow \pi^0 e^+$ , would reveal the gauge group structure at extremely high energy scale.

In SUSY models, on the other hand,  $B$ -violating operators arise due to the flavor

structure of the models rather than the gauge structure. Supersymmetric  $B$ -violating dimension four and five operators are necessarily flavor-off-diagonal due to the Bose symmetry of the superfields. For example, the dimension four operator  $\epsilon^{abc}U_a^c D_b^c D_c^c$  exists only when two down quark  $D^c$  fields belong to different generations. The same is true for dimension five operators such as  $\epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}Q_\alpha^a Q_\beta^b Q_\gamma^c L_\delta$  which will be the main topic in this Letter. Therefore  $B$ -violating operators are presumably intimately related to the physics which generate Yukawa interactions and flavor mixings. Indeed, the dimension five operators are generated in generic SUSY-GUT models via the exchange of color-triplet Higgses, and the coupling constants are those of Yukawa interactions.

Because  $B$  violation can occur through such low dimension operators, as opposed to dimension 6 in original GUTs, SUSY models are sensitive to flavor physics through  $B$  violation all the way up to the Planck scale. For instance, the operator  $\frac{1}{M_p}(Q_2 Q_2)(Q_1 L_i)$  gives a proton lifetime shorter than the experimental bound by about 14 orders of magnitude. Since proton decay is a flavor changing process, further suppression can arise if the operators are suppressed by Yukawa couplings, as is the case when they are induced by Higgs triplet exchange in a GUT. As we discuss below, when the sole source of flavor symmetry breaking are the Yukawa couplings themselves, then the decays  $p \rightarrow K^0 \ell^+$  will be unobservable, being suppressed by the  $u$  quark Yukawa coupling. Alternatively, flavor symmetries broken at a scale  $\Lambda$  can give a suppression of the form  $\Lambda/M_p$  to some power which can both explain the long lifetime of the proton, as well as render the decay modes  $p \rightarrow K^0 \ell^+$  observable at Super-Kamiokande — even in string-inspired models without gauge unification below the string scale.

In the next section, we briefly review the form of dimension five  $B$  violating operators in supersymmetric theories, and discuss their flavor structures. We then show that  $p \rightarrow K^0 \ell^+$  modes are unobservable in conventional GUT-models. Therefore, an observation of these modes would signal a different structure of flavor physics at very short distances. We point out in §4 that operators suppressed by powers of Planck mass can significantly contribute to  $p \rightarrow K^0 \ell^+$  modes with branching fractions  $\sim 0.2$  in certain flavor physics models. We also discuss both upper and lower bounds on the flavor symmetry breaking scale. We conclude the Letter in §5.

## 2. Dimension Five $B$ -Violating Operators

We begin by summarizing the properties of the dimension five  $B$  violating operators in supersymmetry, following the notation of [2]. These operators are either composed entirely out of the weak doublet superfields  $Q$  and  $L$  and take the form  $\Delta\mathcal{B} = QQQ L$ , or else involve only the weak singlet superfields,  $\Delta\mathcal{B}' = U^c U^c D^c E^c$ . Because of antisymmetrization in color, at least two families must be involved; to avoid suppression by small mixing angles, we need only consider the quark superfields from the lightest two families. The singlet operators  $\Delta\mathcal{B}'$  make a much smaller contribution to proton decay and we will ignore them, since they require at least one power each of the  $c$  quark and  $\tau$  lepton Yukawa couplings to convert the right-handed charm quark superfield into a lighter flavor. The  $\Delta\mathcal{B}$  operators have two types of flavor structure for the quarks:

$$\begin{aligned}\Delta\mathcal{B}_{\{1,i\}} &= (Q_1 Q_1)(Q_2 L_i) = 2\epsilon_{abc}(U_a D'_b)(C_c E_i - S'_c N_i) , \\ \Delta\mathcal{B}_{\{2,i\}} &= (Q_2 Q_2)(Q_1 L_i) = 2\epsilon_{abc}(C_a S'_b)(U_c E_i - D'_c N_i) .\end{aligned}\tag{2.1}$$

In the above equation we take  $Q$  to be the  $SU(2)$  eigenstates  $Q = (U, D') = (U, VD)$ , where  $U$  and  $D$  are mass eigenstates and  $V$  is the CKM matrix. The bracketed quantities are  $SU(2)$  singlets, and  $i$  refers to the lepton family, with  $i = 1, 2$  for the charged leptons  $E$  or  $1, 2, 3$  for the neutrinos  $N$ . Operators of the form  $(Q_1 Q_2)(Q_2 L_i)$  and  $(Q_1 Q_2)(Q_1 L_i)$  can be Fierz rearranged into the above forms.

The operators (2.1) may be dressed with gauginos to yield the four-fermion operators in the low energy, nonsupersymmetric theory that are relevant for proton decay. There are five types of operators relevant for proton decay; two that result in a positively charged lepton in the final state:

$$\mathcal{O}_{1i} = (su)(ue_i) \quad \mathcal{O}_{2i} = (du)(ue_i) ,\tag{2.2}$$

where  $i = 1, 2$ , and three operators giving rise to antineutrinos in the final state:

$$\mathcal{O}_{3i} = (du)(s\nu_i) \quad \mathcal{O}_{4i} = (su)(d\nu_i) \quad \mathcal{O}_{5i} = (du)(d\nu_i) ,\tag{2.3}$$

where  $i = 1, 2, 3$ . Our notation is such that

$$(su)(ue_i) \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}(s_L^{a\alpha}u_L^{b\beta})(u_L^{c\gamma}e_{Li}^\delta) ,$$

etc. Assuming that the squarks are nearly degenerate, gluinos do not contribute, while wino dressing at one loop yields

$$\Delta\mathcal{B}_{\{1,i\}} \rightarrow \frac{\alpha_2}{\pi} \sum_{n=1}^5 a_{ni} \mathcal{O}_{ni} , \quad \Delta\mathcal{B}_{\{2,i\}} \rightarrow \frac{\alpha_2}{\pi} \sum_{n=1}^5 b_{ni} \mathcal{O}_{ni} , \quad (2.4)$$

where  $\alpha_2 = \alpha/\sin^2 \theta_w$  and

$$\begin{aligned} a_{1i} &= -\cos \theta_c [f(c, d') + f(\nu_i, d')] & b_{1i} &= 0 \\ a_{2i} &= -\tan \theta_c a_{1i} & b_{2i} &= 0 \\ a_{3i} &= -\cos^2 \theta_c [f(c, e_i) + f(u, d')] & b_{3i} &= -\cos \theta_c \sin \theta_c [f(c, e_i) + f(c, d')] \\ a_{4i} &= -\tan^2 \theta_c a_{3i} & b_{4i} &= b_{3i} \\ a_{5i} &= -\tan \theta_c a_{3i} & b_{5i} &= -\tan \theta_c b_{3i} . \end{aligned} \quad (2.5)$$

The function  $f$  is given by [3]

$$f(u, d) \equiv \frac{m_{\tilde{w}}}{m_{\tilde{u}}^2 - m_{\tilde{d}}^2} \left( \frac{m_{\tilde{u}}^2}{m_{\tilde{u}}^2 - m_{\tilde{w}}^2} \ln \frac{m_{\tilde{u}}^2}{m_{\tilde{w}}^2} - \frac{m_{\tilde{d}}^2}{m_{\tilde{d}}^2 - m_{\tilde{w}}^2} \ln \frac{m_{\tilde{d}}^2}{m_{\tilde{w}}^2} \right) . \quad (2.6)$$

For degenerate squarks with mass  $m_{\tilde{q}} \gg m_{\tilde{w}}$ ,  $f$  is simply given by  $f \simeq m_{\tilde{w}}/m_{\tilde{q}}^2$ .

We can quantify the effects of the operators (2.1) by parametrizing their strength in the superpotential  $W$  in terms of the modified Planck scale

$$M_p^\star \equiv \frac{M_p}{\sqrt{8\pi}} = 2.4 \times 10^{18} \text{ GeV} \quad (2.7)$$

and the dimensionless coupling constants  $g_{1,2}$ :

$$W_5 = \frac{1}{M_p^\star} \sum_{i=1}^3 [g_{1i} \Delta\mathcal{B}_{\{1,i\}} + g_{2i} \Delta\mathcal{B}_{\{2,i\}}] . \quad (2.8)$$

In terms of these  $g$  coefficients one finds the widths [2]

$$\begin{aligned} \Gamma(p \rightarrow K^0 \ell_i^+) &= \left( \frac{\mathcal{A} \beta \alpha_2 \cos \theta_c}{\pi M_p^\star} \right)^2 \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2} \left| g_{1i} \kappa_1 [f(c, d') + f(\nu_i, d')] \right|^2 \\ \Gamma(p \rightarrow K^+ \bar{\nu}_i) &= \left( \frac{\mathcal{A} \beta \alpha_2 \cos \theta_c}{\pi M_p^\star} \right)^2 \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2} \\ &\quad \times \left| g_{1i} \kappa_2 \cos \theta_c [f(c, e_i) + f(u, d')] + g_{2i} \kappa_3 \sin \theta_c [f(c, e_i) + f(c, d')] \right|^2 \\ \Gamma(n \rightarrow K^0 \bar{\nu}_i) &= \left( \frac{\mathcal{A} \beta \alpha_2 \cos \theta_c}{\pi M_p^\star} \right)^2 \frac{(m_n^2 - m_K^2)^2}{32\pi m_n^3 f_\pi^2} \\ &\quad \times \left| g_{1i} \kappa_2 \cos \theta_c [f(c, e_i) + f(u, d')] + g_{2i} \kappa_4 \sin \theta_c [f(c, e_i) + f(c, d')] \right|^2 \end{aligned} \quad (2.9)$$

where  $\beta$  is an unknown strong matrix element, estimated to be  $\sim 10^{-2} \text{ GeV}^3$ , while the  $\kappa$ 's are coefficients computable from the QCD chiral Lagrangian in terms of the axial current matrix elements  $D$  and  $F$  [4], [2]:

$$\begin{aligned}
\kappa_1 &\equiv 1 - \frac{m_p}{m_\Lambda}(D - F) = 0.70 \\
\kappa_2 &\equiv 1 + \frac{m_p}{3m_\Lambda}(D + 3F) = 1.6 \\
\kappa_3 &\equiv 1 + \frac{m_p}{m_\Lambda}(D + F) = 2.0 \text{ .} \\
\kappa_4 &\equiv 2 + \frac{2m_p}{m_\Lambda}F = 2.7 \text{ .}
\end{aligned}
\tag{2.10}$$

The above formulae include one loop scaling effects due to the gauge interactions from  $M_p^*$  down to 1 GeV, which give an enhancement of  $\mathcal{A} \simeq 10.5$  in the amplitude. Again, note that  $\Delta\mathcal{B}_{\{1,i\}}$  makes the sole contribution to  $p \rightarrow K^0 \ell_i^+$ ; furthermore, the contribution of  $\Delta\mathcal{B}_{\{2,i\}}$  to  $p \rightarrow K^+ \bar{\nu}_i$  is Cabbibo suppressed. Since  $\kappa_2 > \kappa_1$  the most stringent experimental limits on the  $g$  couplings come from [5]

$$\Gamma(p \rightarrow K^+ \bar{\nu}) < (1.0 \times 10^{32} \text{ yr})^{-1},$$

$$\Gamma(n \rightarrow K^0 \bar{\nu}) < (8.6 \times 10^{31} \text{ yr})^{-1},$$

which yield

$$\begin{aligned}
\sqrt{\sum_i |g_{1i}|^2} &< 3.6 \times 10^{-8} \times \left( \frac{1 \text{ TeV}^{-1}}{f(c, d') + f(u, \ell_i)} \right) \left( \frac{0.01 \text{ GeV}^3}{\beta} \right) \text{ ,} \\
\sqrt{\sum_i |g_{2i}|^2} &< 1.0 \times 10^{-7} \times \left( \frac{1 \text{ TeV}^{-1}}{f(c, d') + f(c, \ell_i)} \right) \left( \frac{0.01 \text{ GeV}^3}{\beta} \right) \text{ .}
\end{aligned}
\tag{2.11}$$

The charged lepton modes are in general smaller than similar neutrino modes because (i)  $g_{2i}$  only contributes to the neutrino modes, and (ii) the  $g_{1i}$  contributions are larger for neutrino modes than for charged lepton modes since the ratio of the amplitudes contains a factor of  $\kappa_1/\kappa_2 = 0.4$ . As we discuss in the following section, in conventional GUT models  $g_{1i} \ll g_{2i}$  and the charged lepton modes are invisible.

### 3. Predictions from GUT models

Consider a generic GUT theory where the only breaking of the chiral flavor symmetries is due to Yukawa interactions with the same general size and texture of the Yukawa couplings in the standard model at low energy, or smaller. This includes GUTs with non-minimal Higgs field content, such as the Georgi-Jarlskog model [6], for example. It follows that all of the color triplet scalars will have couplings of the form  $Q\tilde{Y}Q$  and  $Q\tilde{Y}'L$ , where  $\tilde{Y} \lesssim Y_U$  and  $\tilde{Y}' \lesssim Y_D$  (or similarly with the  $U$  and  $D$  subscripts reversed, in the case of a “flipped” charge embedding). In such theories the  $\Delta\mathcal{B}_{\{1,i\}}$  operators are generated with strength  $\tilde{Y}_{11}$  ( $\tilde{Y}'_{2i}$ ) while the  $\Delta\mathcal{B}_{\{2,i\}}$  operators are generated with strength  $\tilde{Y}_{22}$  ( $\tilde{Y}'_{1i}$ ). As such, one sees that the  $\Delta\mathcal{B}_{\{1,i\}}$  operators are suppressed relative to  $\Delta\mathcal{B}_{\{2,i\}}$  by  $\sim \sqrt{m_u/m_c}$ . Therefore the charged lepton decay modes of the proton are unlikely to be seen.

For example, in minimal  $SU(5)$  [2], colored Higgs exchange generates operators of the form

$$\frac{1}{2M_{H_c}} y_{u_i} y_{d_k} V_{jk}^* (Q_i Q_i) (Q_j L_k) . \quad (3.1)$$

Operators involving third generation quark fields can be at most comparable to those with first two generation fields only in (3.1), only if one takes extremal values for CKM angles in the range presently allowed by experiments. Therefore hereafter we only discuss operators involving quark fields from the first two generations, the relevant terms being:

$$W_5 \sim \frac{1}{2M_{H_c}} [y_u y_{d_i} V_{2i}^* \Delta\mathcal{B}_{\{1,i\}} + y_c y_{d_i} V_{1i}^* \Delta\mathcal{B}_{\{2,i\}}] . \quad (3.2)$$

Evidently only  $\Delta\mathcal{B}_{\{2,2\}}$  is relevant for proton decay since the other operators are suppressed by an additional power of  $\sin\theta_c$ ,  $y_u/y_c$ , or both. Since the  $p \rightarrow K^0 \ell^+$  decay can only proceed through  $\Delta\mathcal{B}_{\{1,1\}}$  and  $\Delta\mathcal{B}_{\{1,2\}}$ , one finds that  $BR(K^0 \ell^+)/BR(K^+ \bar{\nu})$  contains a suppression factor of  $(y_u \kappa_1 / y_c \kappa_3 \sin^2 \theta_c)^2 \sim 6 \times 10^{-4}$ . In the more realistic GUT example of ref. [7], the effective superpotential for proton decay is [8]

$$W_5 \sim \frac{y_c y_s}{2M_{H_c}} \frac{50\gamma y_s}{y_c} \sqrt{\frac{y_d}{y_s}} \left[ \delta \frac{y_u}{y_c} \Delta\mathcal{B}_{\{1,1\}} + \delta \sqrt{\frac{y_u}{y_c}} \Delta\mathcal{B}_{\{2,1\}} + \sqrt{\frac{y_u}{y_c}} \Delta\mathcal{B}_{\{1,2\}} + \Delta\mathcal{B}_{\{2,2\}} \right] , \quad (3.3)$$

where the coefficients  $\gamma \sim 1$ ,  $\delta \sim 0.5$  are combinations of model dependent Clebsch–Gordon factors. The suppression of charged lepton decay modes is an order of magnitude less severe

than in minimal  $SU(5)$ . Nevertheless, one finds from eqs. (2.8)-(2.10) that for this model

$$\begin{aligned} \frac{BR(p \rightarrow K^0 \mu^+)}{BR(p \rightarrow K^+ \bar{\nu})} &\simeq 8 \times 10^{-3} , \\ \frac{BR(p \rightarrow K^0 e^+)}{BR(p \rightarrow K^+ \bar{\nu})} &\simeq 6 \times 10^{-6} , \end{aligned} \quad (\text{model of ref. [7]}) \quad (3.4)$$

and so  $p \rightarrow K^0 \mu^+$  remains unlikely to be seen, while  $p \rightarrow K^0 e^+$  is still certainly undetectable.

In a “flipped” GUT the roles played by the up and down Yukawa couplings is effectively reversed, and the quark doublet  $Q = (U, D')$  is replaced by  $Q' = (U', D) = V^\dagger Q$ . Furthermore, the lepton doublet no longer involves mass eigenstates for the charged leptons, and is given by  $L' = (\nu, \ell') = (\nu, V_\ell \ell)$ , where  $V_\ell$  is the lepton analogue of the CKM matrix  $V$ . Thus Eq. (3.1) is replaced by

$$\frac{1}{2M_{H_c}} y_{d_i} y_{u_k} V_{kj} (Q'_m Q'_n) (Q'_j L'_k) . \quad (3.5)$$

With these interactions one finds the dominant contribution to  $p \rightarrow K^+ \nu_\tau$  to be of the form  $y_d y_t V_{ts} (Q'_1 Q'_1) (Q'_2 L'_3)$ , without the further accompanying  $V_{ts}$  suppression found in the minimal  $SU(5)$  example. If one assumes that  $V_\ell$  is close to the unit matrix, then one finds that  $p \rightarrow K^0 \mu^+$  generated by  $y_s y_c V_{cd} (Q'_2 Q'_2) (Q'_1 L'_2)$  dominates the charged lepton modes, but at a rate smaller than  $p \rightarrow K^+ \bar{\nu}$  by a factor of

$$\frac{BR(p \rightarrow K^0 \ell^+)}{BR(p \rightarrow K^+ \bar{\nu})} \sim \left( \frac{m_s m_c V_{cd} V_{us}^* \kappa_1}{m_d m_t V_{ts} \kappa_2} \right)^2 \simeq 8 \times 10^{-3} \quad (\text{flipped } SU(5), V_\ell \sim 1) \quad (3.6)$$

so that it is still invisible.<sup>1</sup> Furthermore, in flipped  $SU(5)$  it is possible to make the parameter  $M_{H_c}$  be effectively much larger than the GUT scale, or infinite [9] so that all proton decay modes are invisible.

It follows then that observing the decay  $p \rightarrow K^0 \ell^+$  (and  $p \rightarrow K^0 e^+$  most strikingly) will be a signal for flavor physics with a structure different than the low energy Yukawa couplings. It is easy to invent an unnatural example of such a model — for example,

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<sup>1</sup> If  $V_\ell$  has large off-diagonal elements (e.g.  $(V_\ell)_{\nu_\tau e} \simeq 1$ ), then the above ratio can be as large as  $\mathcal{O}(1)$ , and the theory is an example of how first family chiral symmetries being badly broken by new flavor physics; in this case the new flavor physics is in the lepton sector.



consider  $SU(5)$  with an extra pair of superfields  $\phi, \bar{\phi}$  transforming as  $5, \bar{5}$  respectively, with a GUT scale mass and a coupling to fermions of the form

$$f_{ij}^u \phi 10_i 10_j + f_{ij}^d \bar{\phi} 10_i \bar{5}_j, \quad (3.7)$$

where the  $f$  couplings have no hierarchical structure and are small enough to be consistent with experimental bounds on the proton lifetime. In such a model the operators  $\Delta\mathcal{B}_{\{1,i\}}$  would be generated with the same strength as  $\Delta\mathcal{B}_{\{2,i\}}$ , and one might see  $p \rightarrow K^0 \ell^+$  — precisely because the  $f$  matrices break the chiral symmetries for the light families in a different (bigger) way than do the Yukawa couplings. This example is illustrative but not interesting, since it renders the structure of the Yukawa coupling inexplicable instead of merely mysterious. In the next section we point out how a model with softly broken flavor symmetries which explain the structure of the Yukawa couplings can also give rise to detectable  $p \rightarrow K^0 \ell^+$  decay.

#### 4. Proton Decay from Planck Scale Physics

For simplicity we will first consider the models without grand unification with a flavor symmetry  $G_f$  broken by the expectation values of some fields  $X$ . The flavor symmetry breaking is assumed to be transmitted to the quarks and leptons by means of particles with mass  $M$  so that the low energy Yukawa couplings are constructed out of powers of  $\epsilon \equiv \langle X \rangle / M$  with a texture dictated by  $G_f$  symmetry [10]. Such models can explain the gross hierarchical features of the quark and lepton mass matrices, such as in the recent examples [11-14]. It is preferred, moreover, that a theory which explains the quark mass matrix also explains at the same time why the squarks do not give rise to large FCNC. Note that one cannot ensure squark degeneracy when  $G_f$  is only an abelian symmetry. It is possible to remedy this with a clever choice of abelian symmetries, as in [11], where nondegenerate squarks have mass matrices that align with the quark masses; a more automatic solution is to enforce degeneracy by having a nonabelian symmetry  $G_f$  throughout the theory, with at least the first two families in an irreducible representation [15], [12,13]

We will take the point of view that an explanation for the lack of FCNC and for the structure of the quark and lepton mass matrices mandates some sort of flavor symmetry,

most likely nonabelian, at short distances. What we point out here is that the flavor symmetries can also control other flavor changing effects, particularly in proton decay. In particular, the dimension five operators (2.8) may be forbidden by  $G_f$  symmetry, while the dimension six operator

$$\frac{g'}{(M_p^*)^2} QQQ LX \quad (4.1)$$

is allowed, in which case one might expect Planck scale physics to generate it with some  $\mathcal{O}(1)$  coupling  $g'$ . Such is the case with the nonabelian discrete symmetry  $\Delta(75)$  discussed in ref. [13]; there the three families of  $Q$  and  $L$  transform in the fundamental triplet representation  $T_1$ , while  $X$  is in the representation  $T_2$  and acquires a  $Z_3$  preserving vacuum expectation value  $\langle X \rangle = (\Lambda, \Lambda, \Lambda)$ . One finds that  $\Delta(75)$  symmetry forbids the dimension five operators  $QQQL$ , while there are several different invariants of the form (4.1). Below the  $\Delta(75)$  symmetry breaking scale  $\Lambda$ , this results in an effective superpotential of the form (2.8) with coefficients

$$g_{1i} \simeq g_{2i} \simeq \frac{g' \Lambda}{M_p^*} . \quad (4.2)$$

Proton decay through the dimension five operators becomes naturally suppressed, without being eliminated entirely, as would be the case if one imposed the anomaly free symmetry  $\exp(2\pi i B/3)$  [16]. Furthermore, the  $\Delta \mathcal{B}_{\{1,i\}}$  operator dominates because of Cabibbo suppression in the  $b$  coefficients of eq. (2.5), and so one finds

$$BR(p \rightarrow K^0 \ell^+) / BR(p \rightarrow K^+ \bar{\nu}) \simeq (\kappa_1 / \kappa_2)^2 = 0.2 . \quad (4.3)$$

Since  $p \rightarrow K^+ \bar{\nu}$  is the dominant decay mode, it follows that if proton decay is seen at all, it will probably be possible to see the charged lepton decay mode. It should be added that the charged lepton modes are easier to reconstruct than the neutrino modes in the Čerenkov detectors, especially in the  $p \rightarrow K^0 \ell^+$ ,  $K^0 \rightarrow \pi^0 \pi^0 \rightarrow \gamma \gamma \gamma \gamma$  final state.

Since the effects of the operator get larger as  $\langle X \rangle$  gets larger, there is now an experimental *upper* bound for the scale of  $G_f$  symmetry breaking  $\Lambda$ :

$$g' \Lambda \lesssim 9 \times 10^{10} \text{ GeV} \quad (\text{dim. } 6), \quad (4.4)$$

with the same uncertainties given in (2.11). This bound is weakened if for a different  $G_f$  symmetry the dimension six operator (4.1) was forbidden (*e.g.*, by an additional  $U(1)$  or

discrete flavor symmetry) and only a dimension 7 operator was allowed; then one would find

$$\sqrt{g'}\Lambda \lesssim 5 \times 10^{14} \text{ GeV} \quad (\text{dim. 7}). \quad (4.5)$$

If  $g'\Lambda$  or  $\sqrt{g'}\Lambda$  are near the upper bounds (4.4), (4.5), then we might hope to see  $p \rightarrow K^0 \ell^+$  at Super-Kamiokande; so what can be said about this scale  $\Lambda$ ? One can derive a crude lower bound on this scale by considering the running of the gauge coupling constants. Models of the sort considered here have the quark and lepton masses arise from a “see-saw” mechanism, whereby the light particles mix with Dirac counterparts of mass  $M$  through the Higgs doublets and the symmetry breaking field  $X$ . Typically  $\Lambda/M \sim 10^{-1}$  in order to explain the size of the observed mixing angles and mass ratios. It follows that there is a fermion contribution to the  $\beta$  functions which is at least three times the usual one, and this typically renders the gauge forces asymptotically un-free above the scale  $M$ ; one can then derive a lower bound on  $M$  — and hence  $\Lambda$  — by requiring that all Landau poles be above the Planck scale. For example, in the un-unified standard model with an additional three Dirac families with mass  $M$ , the one-loop  $\beta$  function for hypercharge leads to a Landau pole below the scale  $M_p^*$  if  $M$  is below  $6 \times 10^{13}$  GeV. Thus for a flavor symmetry breaking scale  $\Lambda \simeq M/20$ , as in ref. [13], avoiding a Landau pole implies the lower bound

$$\Lambda \gtrsim 3 \times 10^{12} \text{ GeV} , \quad (4.6)$$

a value that is easily consistent with the upper bound (4.5), but not with (4.4) assuming that  $g' = \mathcal{O}(1)$ . This suggests that proton decay operators in such models must be protected by flavor symmetries, at least through dimension six. Dimension seven operators of the form (4.1), with an extra factor of  $X/M_p^*$  are allowed so long as (4.6) and (4.5) are satisfied. The Super-Kamiokande experiment is expected to improve bounds on the proton decay by a factor of  $\sim 30$ , which would be sensitive to dimension seven operators with  $\Lambda$  lower than the bound (4.5) by a factor of  $\sim 5 - 6$ . Thus it is possible, but not necessary, that the effects of dimension seven operators could be detected if close to the bound (4.5). We note in passing that at the bound (4.5),  $\Lambda \sim 5 \times 10^{14}$  GeV, then the ratio  $\Lambda/M_{GUT} \sim 1/20$  could conveniently provide the small parameter used in constructing the

fermion hierarchy.

If the gauge groups are enlarged or unified below  $M_p^\star$  the lower bound (4.6) on  $\Lambda$  might be weakened somewhat, leaving a small window for dimension six operators, which could be detected at Super-Kamiokande. However, extended theories typically require many new matter fields in the Higgs sector which can completely cancel the gauge contribution to the 1-loop  $\beta$  function. Therefore using unification to lower the bound (4.6) is not easy to do.

## 5. Conclusions

We have shown that detecting the decay mode  $p \rightarrow K^0 \ell^+$  would be a certain signal for new flavor physics at short distances in a supersymmetric theory. Such decay modes can be dominant if the dangerous dimension 5 proton decay operators generated at the Planck scale are forbidden only by flavor symmetries which allow higher dimension  $B$  violating operators. Unlike other probes for new flavor symmetries, these effects become stronger as the flavor scale  $\Lambda$  becomes *higher*. In principle such operators could be very small for  $\Lambda \ll M_p^\star$ . However for most theories generating fermion masses *a la* Froggatt and Nielsen, the loss of asymptotic freedom at short distances places a lower bound on  $\Lambda$ . The bound turns out to be within a factor of  $\sim 10^2$  of the upper bound on  $\Lambda$  from proton decay, suggesting that the Super-Kamiokande experiment may be able to detect their effects. In such theories, proton decay is not itself a signal for a unification of gauge forces.

It is worth noting in conclusion that in principle nonabelian flavor symmetries can serve not only to explain the quark and lepton masses, squark degeneracy, and the suppression of dimension 5  $B$  violating operators, but may also control the dimension  $\leq 4$   $B$  and  $L$  violating operators. The flavor symmetry  $\Delta(75)$  mentioned in above and in ref. [13] does not eliminate the dimension four  $U^c D^c D^c$  operator (being a subgroup of  $SU(3)$ ), but other symmetries could. In such a model,  $R$ -parity could arise as an accidental symmetry of the renormalizable theory broken only by  $M_p^\star$  suppressed operators, making the  $LSP$  unstable, but with a very long lifetime.<sup>2</sup>

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<sup>2</sup> The strongest constraint from proton decay is  $\lambda_{U^c D^c S^c} \lambda_{Q_1 S^c L_{1,2}} < \mathcal{O}(10^{-26})$ . This suggests

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$\lambda < \mathcal{O}(10^{-13})$ , roughly the square of first generation Yukawa couplings. If this were the typical size of  $R$ -parity violation, then the  $LSP$  would decay near the time of nucleosynthesis, threatening light element abundances. However  $R$ -parity violation due to other operators is constrained only by the cosmological requirement that  $\lambda < \mathcal{O}(10^{-8})$  in order to preserve the baryon asymmetry [17].  $R$ -parity violation at this level allows the  $LSP$  to decay long before nucleosynthesis. The challenge for flavor model building is to satisfy these two constraints simultaneously.

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